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Numerical method for the solution of a malaria model using the collocation method

Ojo Olamiposi Aduroja[1] and Ganiyu Ajileye[2]*

[1]Department of Mathematics, University of Ilesa, Ilesa, Osun State, Nigeria.
[2]Department of Mathematics, Federal University Wukari, Taraba State, Nigeria.

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This work proposes an approximate solution to a malaria transmission model based on power series expansions and the Hermite collocation method. The work presents the malaria dynamics as a system of nonlinear differential equations that describe the susceptible and infected populations among humans and vectors. The model equations are turned into a set of solvable terms using power series techniques, which allows for analytical insight into the solution structure. The Hermite collocation approach is then used as a numerical approximation tool, employing orthogonal basis functions to generate very precise and stable solutions, even for complicated nonlinearities in the malaria model. Numerical simulations employing this integrated technique show convergence and high agreement between the approximate solutions for various parameters. The findings demonstrate the historical evolution of susceptible and infected classes, offering important epidemiological insights into disease progression and control. This hybrid analytical-numerical framework provides an effective and computationally efficient tool for investigating malaria transmission patterns, allowing for more accurate design of public health interventions and vector control tactics.

Key words: Collocation method, approximate solution, hermite polynomial and power series polynomial, malaria model.

INTRODUCTION

Malaria is an infectious disease caused by the Plasmodium parasite that is transferred to humans via the bite of the female Anopheles mosquito (Chitnis et al., 2006; Onah et al., 2019). It is a lethal disease that is particularly prevalent in sub-Saharan African and Asian countries, posing a global hazard. In 2020, it was reported that there are roughly 241 million malaria cases worldwide, with 627,000 fatalities from malaria (World Health Organisation (WHO), 2021). Africa had 95% of cases and 96% of deaths, with children under the age of five accounting for 80% of all deaths (WHO, 2021).

In 2021, approximately half of the world's population was at danger of malaria, with a projected 244 million cases worldwide and 619,000 fatalities (WHO, 2022). Tropical Africa accounted for 92% of cases and 93% of deaths, with Nigeria having the largest number (25%), and Uganda having the lowest (4%). Malaria cases were 249 million worldwide in 2022, with 608,000 deaths (WHO, 2023). In 2022, children under the age of 5 accounted for 76% of all deaths (WHO, 2023). Malaria symptoms

*Corresponding author. E-mail: ajileye@fuwukari.edu.ng.

include fever (high or moderate), shivering chills (severe or moderate), newborn mortality, morbidity, headache, bloody stools, severe anemia, excessive perspiration, nausea, vomiting, stomach pain, muscle pain, diarrhea, convulsions, coma, and other symptoms (Amaoh-Mensah et al., 2018).

Furthermore, exact solutions to most realistic systems of ordinary differential equations cannot be obtained; hence, numerical and approximate methods are required to determine approximate solutions. Many researchers have investigated a variety of strategies for solving ordinary differential equations. Some of these methods are the multi-step method proposed by (Hojjati et al., 2004), the collocation method presented by (Mastorakis, 2005; Ajileye et al., 2022), the Adomian decomposition method by Shawagfeh and Kaya (2004), the exponential Galerkin method introduced by Yuzbasi and Karacayir (2017), the exponential collocation method proposed by Yüzba,sı (2016), the Galerkin finite element method by Al-Omari et al. (2013), the Bernoulli wavelets by Rahimkhani and Ordokhsani (2019), Bernstein polynomial for solving fractional differential equations by Asgari and Ezzati (2017), Mirzaee and Samadyar (2019), Differential Transformation method by (Ibrahim and Ismail, 2017), Laplace-Adomian decomposition method by (Dogan and Akin (2012), homotopy perturbation method by Rafe et al. (2007), Sinc collocation method by John et al. (2024), Modified simpson's rule by Djaidja and Khirani (2024), Fractional Differential Equation (FDE) method by Osu et al. (2025) and many more. Hermite Polynomials are formed by repeatedly taking derivatives with scaling factors and have an important property known as orthogonality, which states that the product of two unique polynomials integrates to zero. This orthogonality facilitates complex calculations in physics and other sciences, while the recurrence relation enables higher-order polynomials to be constructed from lower-order ones (Gulsu et al., 2011). This paper investigates the application of the collocation method to obtain approximate solutions for a malaria model.

MATERIALS AND METHODS

Mathematical model formulation

The mathematical model for malaria presented here has five compartments made up of human and mosquito populations. The model includes a human population $N_h(t)$ and a mosquito population $N_m(t)$ at time t. The human population at time t is partitioned into sub-populations namely: susceptible humans $S_h(t)$, infected humans $I_h(t)$, recovered humans $R_h(t)$. The mosquito population $N_m(t)$ at time t is also partitioned into susceptible mosquitoes $S_m(t)$ and infected mosquitoes $I_m(t)$. The model in Equation 1 describing the transmission dynamics of malaria are given by:

$$\frac{dS_h}{dt} = \Lambda_h - \beta_h \frac{S_h I_m}{N_h} - \mu_h S_h$$
$$\frac{dI_h}{dt} = \beta_h \frac{S_h I_m}{N_h} - (\gamma_h + \mu_h + \delta_h) I_h$$
$$\frac{dR_h}{dt} = \gamma_h I_h - \mu_h R_h$$
$$\frac{dS_m}{dt} = \Lambda_m - \beta_m \frac{S_m I_h}{N_h} - \mu_m S_m$$
$$\frac{dI_m}{dt} = \beta_m \frac{S_m I_h}{N_h} - \mu_m I_m \qquad (1)$$

with initial conditions in Equation 2

$$S_h(0) = N_{s_h},\ I_h(0) = N_{I_h},\ R_h(0) = N_{R_h},\ S_m(0) = N_{s_m},\ I_m(0) = I_{s_m} \qquad (2)$$

Where, $\Lambda_h\ \Lambda_m$: recruitment rate of humans and mosquitoes, $\mu_h\ \mu_m$: natural death rates, $\beta_h\ \beta_m$: transmission probabilities, γ_h: recovery rate of humans, δ_h: disease-induced death rate of humans.

The total human and mosquito population are assumed constant

$N_h = S_h(t) + I_h(t) + R_h(t)$ and $N_m = S_m(t) + I_m(t)$

Analysis of the model

Disease-Free Equilibrium (DFE)

The existence of the Malaria-free equilibrium state of the model was investigated. At the DFE, no infection persist so in Equation 3

$$\frac{dI_h}{dt} = 0, \frac{dR_h}{dt} = 0, \frac{dI_m}{dt} = 0 \qquad (3)$$

The system reduce to (Equation 4)

$$0 = \Lambda_h - \mu_h S_h' \ \rightarrow\ S_h' = \frac{\Lambda_h}{\mu_h}$$
$$0 = \Lambda_m - \mu_m S_m' \ \rightarrow\ S_m' = \frac{\Lambda_m}{\mu_m} \qquad (4)$$

The DFE is shown in Equation 5

$$E_0 = \left(\frac{dS_h}{dt}, \frac{dI_h}{dt}, \frac{dR_h}{dt}, \frac{dS_m}{dt}, \frac{dI_m}{dt}\right) = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_m}{\mu_m}\right) \qquad (5)$$

Basic reproduction number

The basic reproduction number for the system is denoted by R_O. It is an important parameter that is used to study the behavior of model. It is defined as the average number of secondary infections infected by an infective individual during an infective period provided that all members of the population are susceptible. We apply the next generation matrix method to obtain the reproduction number.

We write $x' = F(x) - V(x)$

Where, F- contains new infections, V- contains transfer (progression, recovery, death).

New infections for the infected compartments (linearized about DFE in Equation 6):

i) For I_h: new infection term $F_1 = \beta_h \frac{S_h I_m}{N_h}$

ii) Linearized at DFE: $F_1 \approx \beta_h \frac{S_h'}{N_h} I_m$

iii) For I_m: new infection term $F_2 = \beta_m \frac{S_m I_h}{N_h}$

iv) Linearized at DFE: $F_2 \approx \beta_m \frac{S_m'}{N_h} I_h$

The Jacobian of F at DFE is

$$F = \begin{pmatrix} 0 & \beta_h \frac{S_h'}{N_h} \\ \beta_m \frac{S_m'}{N_h} & 0 \end{pmatrix} \tag{6}$$

Transition (outflows) from infected compartments

i) For I_h: transfer term $V_1 = (\gamma_h + \mu_h + \delta_h) I_h$

ii) For I_m: transfer term $V_2 = \mu_m I_m$

Hence, the Jacobian V at DFE is shown in Equation 7

$$V = \begin{pmatrix} \gamma_h + \mu_h + \delta_h & 0 \\ 0 & \mu_m \end{pmatrix} \tag{7}$$

The next-generation matrix is $K = FV^{-1}$ in Equation 8

$$K = \begin{pmatrix} 0 & \frac{\beta_h S_h'}{N_h \mu_m} \\ \frac{\beta_m S_m'}{N_h(\gamma_h + \mu_h + \delta_h)} & 0 \end{pmatrix} \tag{8}$$

The spectral radius (dominant eigenvalue in magnitude) is $\rho(k)$ in Equation 9

$$R_0 = \rho(k) = \sqrt{\frac{\beta_h S_h'}{N_h \mu_m} \cdot \frac{\beta_m S_m'}{N_h(\gamma_h + \mu_h + \delta_h)}} \tag{9}$$

Using $S_h' = N_h$ at the DFE and set $S_m' = N_m$ in Equation 10

Hence

$$R_0 = \sqrt{\frac{\beta_h \beta_m N_m}{N_h \mu_m (\gamma_h + \mu_h + \delta_h)}} \tag{10}$$

Therefore, the disease-free equilibrium point is locally asymptotically stable when $R_0 < 1$, otherwise it is unstable.

Approximate solution of the malaria model using power series collocation method

Each compartment was approximated $y(t) \in (S_h, I_h, R_h, S_m, I_m) on\ t \in [0.T]$ by a polynomial of degree n.

$$y(t) \approx y_n(t) = \sum_{j=0}^{N} a_j t^j = \sum_{j=0}^{N} a_j \varphi_j(t)$$

where φ_j is the monomial (t^j for power series) and $a_j's$ are the parameters coefficient to be determined.

Explicit matrix assembly for malaria compartment

Let the solution of Equation 11 be approximated by

$$y(t) = \sum_{j=0}^{N} a_j t^j = \varphi(t)\boldsymbol{A} \tag{11}$$

where

$$\varphi(t) = [\varphi_0(t) \quad \varphi_1(t) \quad \varphi_2(t) \cdots \varphi_N(t)]$$

And

$$\boldsymbol{A} = [a_0 \quad a_1 \quad a_2 \cdots a_N]$$

For S_h: (12)

Differentiate Equation 13 gives

$$y'(t) = \sum_{j=0}^{N} a_j j t^{j-1} = \varphi'(t)\boldsymbol{A} \tag{13}$$

Re-write Equation 14 as

$$S_h' + \mu_h S_h = \Lambda_h - \beta_h \frac{S_h I_m}{N_h} \tag{14}$$

Substitute Equations 15 and 16 and 17 gives

$$[\varphi'(t) + \mu_h \varphi(t)]\boldsymbol{A} = \Lambda_h - \frac{\beta_h}{N_h}(\varphi(t)^2)\boldsymbol{A}$$
$$\left[\varphi'(t) + \mu_h \varphi(t) - \frac{\beta_h}{N_h}(\varphi(t)^2)\right]\boldsymbol{A} = \Lambda_h \tag{15}$$

Equation (16) can be writing as

$$K(t)\boldsymbol{A} = f(t) \tag{16}$$

where

$$K(t) = \varphi'(t) + \mu_h \varphi(t) - \frac{\beta_h}{N_h}(\varphi(t)^2) \text{ and } f(t) = \Lambda_h$$

Using standard collocation points on Equation 17 gives

$$t_i = a + \frac{b-a}{N} i, \quad i = 1, 2, \cdots N$$
$$K(t_i)\boldsymbol{A} = f(t_i) \tag{17}$$

Where

$$K(x_i) = \begin{bmatrix} K_0(t_0) & K_1(t_0) & K_2(t_0) & \Lambda & K_N(t_0) \\ K_0(t_1) & K_1(t_1) & K_2(t_1) & \Lambda & K_N(t_1) \\ M & M & M & M & M \\ K_0(t_N) & K_1(t_N) & K_2(t_N) & \Lambda & K_N(t_N) \end{bmatrix}, f(t_i) = \begin{bmatrix} f(t_0) \\ f(t_1) \\ M \\ f(t_N) \end{bmatrix}$$

For I_h:

Re-write Equation 18 as

$$I_h' + CI_h = \beta_h \frac{S_h I_m}{N_h} \tag{18}$$

where $C = \gamma_h + \mu_h + \delta_h$

Substitute Equations 19, 20 and 21 gives

$$[\varphi'(t) + C\varphi(t)]\boldsymbol{A} = \frac{\beta_h}{N_h}(\varphi(t)^2)\boldsymbol{A}$$
$$\left[\varphi'(t) + C\varphi(t) - \frac{\beta_h}{N_h}(\varphi(t)^2)\right]\boldsymbol{A} = 0 \tag{19}$$

Equation 20 can be writing as

$$Q(t)\boldsymbol{A} = w(t) \tag{20}$$

where

$$Q(t) = \varphi'(t) + C\varphi(t) - \frac{\beta_h}{N_h}(\varphi(t)^2) \text{ and } w(t) = 0$$

Using standard collocation points on Equation 21 gives

$$t_i = a + \frac{b-a}{N} i, \quad i = 1, 2, \cdots N$$
$$Q(t_i)\boldsymbol{A} = w(t_i) \tag{21}$$

Where

$$Q(x_i) = \begin{bmatrix} Q_0(t_0) & Q_1(t_0) & Q_2(t_0) & \Lambda & Q_N(t_0) \\ Q_0(t_1) & Q_1(t_1) & Q_2(t_1) & \Lambda & Q_N(t_1) \\ M & M & M & M & M \\ Q_0(t_N) & Q_1(t_N) & Q_2(t_N) & \Lambda & Q_N(t_N) \end{bmatrix}, w(t_i) = \begin{bmatrix} w(t_0) \\ w(t_1) \\ M \\ w(t_N) \end{bmatrix}$$

For R_h:

Re-write Equation 22 as

$$R_h' + \mu_h R_h = \gamma_h I_h \tag{22}$$

Substitute Equations 23, 24 and 25 gives

$$[\varphi'(t) + \mu_h \varphi(t)]\boldsymbol{A} = \gamma_h \varphi(t)\boldsymbol{A}$$
$$[\varphi'(t) + \mu_h \varphi(t) - \gamma_h \varphi(t)]\boldsymbol{A} = 0 \tag{23}$$

Equation 24 can be writing as

$$M(t)\boldsymbol{A} = j(t) \tag{24}$$

where

$$M(t) = \varphi'(t) + C\varphi(t) - \frac{\beta_h}{N_h}(\varphi(t)^2) \text{ and } w(t) = 0$$

Using standard collocation points on Equation 25 gives

$$t_i = a + \frac{b-a}{N} i, \quad i = 1, 2, \cdots N$$
$$M(t_i)\boldsymbol{A} = j(t_i) \tag{25}$$

where

$$M(x_i) = \begin{bmatrix} M_0(t_0) & M_1(t_0) & M_2(t_0) & \Lambda & M_N(t_0) \\ M_0(t_1) & M_1(t_1) & M_2(t_1) & \Lambda & M_N(t_1) \\ M & M & M & M & M \\ M_0(t_N) & M_1(t_N) & M_2(t_N) & \Lambda & M_N(t_N) \end{bmatrix}, j(t_i) = \begin{bmatrix} j(t_0) \\ j(t_1) \\ M \\ j(t_N) \end{bmatrix}$$

For S_m:

Re-write Equation 26 as

$$S_m' + \mu_m S_m = \Lambda_m - \beta_m \frac{S_m I_h}{N_h} \quad (26)$$

Substitute Equations 27, 28 and 29 gives

$$[\varphi'(t) + \mu_m \varphi(t)]\boldsymbol{A} = \Lambda_m - \frac{\beta_m}{N_h}(\varphi(t)^2)\boldsymbol{A}$$

$$\left[\varphi'(t) + \mu_m \varphi(t) - \frac{\beta_m}{N_h}(\varphi(t)^2)\right]\boldsymbol{A} = \Lambda_m \quad (27)$$

Equation 28 can be writing as

$$G(t)\boldsymbol{A} = z(t) \quad (28)$$

Where

$$G(t) = \varphi'(t) + \mu_m \varphi(t) - \frac{\beta_m}{N_h}(\varphi(t)^2)$$ and

$$z(t) = \Lambda_m$$

Using standard collocation points on Equation 29 gives

$$t_i = a + \frac{b-a}{N} i, \quad i = 1, 2, \cdots N$$

$$G(t_i)\boldsymbol{A} = z(t_i) \quad (29)$$

Where

$$G(x_i) = \begin{bmatrix} G_0(t_0) & G_1(t_0) & G_2(t_0) & \Lambda & G_N(t_0) \\ G_0(t_1) & G_1(t_1) & G_2(t_1) & \Lambda & G_N(t_1) \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ G_0(t_N) & G_1(t_N) & G_2(t_N) & \Lambda & G_N(t_N) \end{bmatrix}, z(t_i) = \begin{bmatrix} z(t_0) \\ z(t_1) \\ \mathrm{M} \\ z(t_N) \end{bmatrix}$$

$$\frac{dI_m}{dt} = \beta_m \frac{S_m I_h}{N_h} - \mu_m I_m$$

For I_m:

Re-write Equation 30 as

$$I_m' + \mu_m I_m = \beta_m \frac{S_m I_h}{N_h} \quad (30)$$

Substitute Equations 31, 32 and 33 gives

$$[\varphi'(t) + \mu_m \varphi(t)]\boldsymbol{A} = \frac{\beta_m}{N_h}(\varphi(t)^2)\boldsymbol{A}$$

$$\left[\varphi'(t) + \mu_m \varphi(t) - \frac{\beta_m}{N_h}(\varphi(t)^2)\right]\boldsymbol{A} = 0 \quad (31)$$

Equation 32 can be writing as

$$P(t)\boldsymbol{A} = v(t)$$
(32)

where

$$P(t) = \varphi'(t) + \mu_m \varphi(t) - \frac{\beta_m}{N_h}(\varphi(t)^2)$$ and $v(t) = 0$

Using standard collocation points on Equation 33 gives

$$P(t) = \varphi'(t) + \mu_m \varphi(t) - \frac{\beta_m}{N_h}(\varphi(t)^2)$$ and $v(t) = 0$ (33)

Where

$$P(x_i) = \begin{bmatrix} P_0(t_0) & P_1(t_0) & P_2(t_0) & \Lambda & P_N(t_0) \\ P_0(t_1) & P_1(t_1) & P_2(t_1) & \Lambda & P_N(t_1) \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ P_0(t_N) & P_1(t_N) & P_2(t_N) & \Lambda & P_N(t_N) \end{bmatrix}, z(t_i) = \begin{bmatrix} v(t_0) \\ c(t_1) \\ \mathrm{M} \\ v(t_N) \end{bmatrix}$$

Approximate solution of the malaria model using hermite collocation method

The Hermite polynomials are identified by

$$H(t) = n! \sum_{m=0}^{M} \frac{(-1)^m}{(n-2m)!}(2t)^{n-2m}, M \in N, \quad 0 \le t \le \infty$$

Each compartment was approximated $y(t) \in (S_h, I_h, R_h, S_m, I_m) on\ t \in [0.T]$ by a polynomial of degree n using Hermite polynomial as basis function.

$$y(x) = \sum_{i=0}^{M} a_i H_i(x) = H(x)\boldsymbol{A}$$

where

$H_i(t) = [H_0(t) \quad H_1(t) \quad \cdots H_M(t)]$ and

$\boldsymbol{A} = [a_0 \ a_1 \cdots a_M]^T$

The same collocation approach was adopted using the Hermite and standard collocation methods to approximate equation (1). Equations (16), (20), (24), (27), and (31) are the algebraic equations that we solve to obtain the values of a's and substitute into the approximate solution to present the numerical solution.

Illustrative example

Application of power series and Hermite polynomial on the malaria model

This section demonstrates the solution method's accuracy and efficiency in the malaria model. The malaria model uses the following parameter values and Maple software was used to perform numerical analyses Table 1 to 4, Figures 1 and 2. Using the baseline parameters:

$$\beta_h = 0.3, \beta_m = 0.2, \tau_h = 0.1, \quad \mu_h = 0.01, \quad \delta_h = 0.01, \quad \mu_m = 0.1,\ N_h = N_m = 1000,$$
$$\Lambda_h = 10,\ \Lambda_m = 100$$

The initial conditions are:

$$S_h(0), I_h(0), R_h(0), S_m(0), I_m(0) = [995, 5, 0, 990, 10]$$

The numerical approximate solution for $\mathrm{N} = 3$ using power series collocation method gives

$$S_h(t) = 990.000000000010 + 1.19799751908289t - 0.598435741248977e - 1t^2 + 0.196924482111172e - 2t^3$$
$$I_h(t) = 4.99999999998244 - .598497861687785t + 0.358615085955116e - 1t^2 - 0.140386529903336e - 2t^3$$
$$R_h(t) = 0$$

$$S_m(t) = 990.000000000010 + 1.19799751908289t - 0.598435741248977e - 1t^2 + 0.196924482111172e - 2t^3$$
$$I_m(t) = 9.99999999999174 - 0.997997933259345t + 0.498529941376802e - 1t^2 - 0.164048942540296e - 2t^3$$

The numerical approximate solutions for $N = 5$ using power series collocation method gives

$$S_h(t) = 995.000000000000 + 0.348500000005487t - 0.174250141208177e - 2t^2 + 0.406660437874962e - 4t^3 - 1.164371497 \times 10^{-7}t^4 - 9.393988876 \times 10^{-8}t^5$$
$$I_h(t) = 5.00000000000000 - 0.598500000202167t + 0.359100514505428e - 1t^2 - 0.149653267840222e - 2t^3 + 0.454387640047571e - 4t^4 + 6.745099768 \times 10^{-7}t^5$$
$$R_h(t) = 0$$
$$S_m(t) = 990.000000000001 + 1.19800000019222t - 0.599000491056358e - 1t^2 + 0.207680300809443e - 2t^3 - 0.524394563399255e - 4t^4 - 9.726791177 \times 10^{-7}t^5$$
$$I_m(t) = 10.0000000000000 - 0.998000000160584t + 0.499000409391215e - 1t^2 - 0.173009142486080e - 2t^3 + 0.436848768003983e - 4t^4 + 8.103279470 \times 10^{-7}t^5$$

The numerical approximate solutions for $N = 3$ using Hermite collocation method gives

$$S_h(t) = 995.0000000 + 0.3484991561t - 0.1741321106e - 2t^2 + 0.7454992859e - 4t^3$$
$$I_h(t) = 5.000000000 - 0.5984808055t + 0.3584296768e - 1t^2 - 0.1454059248e - 2t^3$$
$$R_h(t) = 0$$
$$S_m(t) = 990.0000000 + 1.197978565t - 0.5982355596e - 1t^2 + 0.2038611410e - 2t^3$$
$$I_m(t) = 10.00000000 - 0.9979821436t + 0.4983631792e - 1t^2 - 0.1698275615e - 2t^3$$

The numerical approximate solutions for $N = 5$ using Hermite collocation method gives

$$S_h(t) = 995.0001116 + 0.3486108470t - 0.2189047234e - 2t^2 + 0.2008059710e - 5t^3 + 0.1485022003e - 3t^4 - 4.016119421 \times 10^{-7}t^5$$
$$I_h(t) = 4.997682522 - 0.6008011215t + 0.4517940610e - 1t^2 - 0.3142278728e - 4t^3 - 0.3037794793e - 2t^4 + 0.6284557456e - 5t^5$$
$$R_h(t) = 0$$
$$S_m(t) = 990.0032114 + 1.201193273t - 0.7274509154e - 1t^2 + 0.3689168806e - 4t^3 + 0.4222918448e - 2t^4 - 0.7378337613e - 5t^5$$
$$I_m(t) = 4.997682522 - .6008011215t + 0.4517940610e - 1t^2 - 0.3142278728e - 4t^3 - 0.3037794793e - 2t^4 + 0.6284557456e - 5t^5$$

Table 1. Numerical approximate solutions for $S_h(t), I_h(t), R_h(t), S_m(t)$ and $I_m(t)$ using the Power series collocation method at $N = 3$.

t	$S_h(t)$	$I_h(t)$	$R_h(t)$	$S_m(t)$	$I_m(t)$
0.2	995.0696306	4.881723657	0.000000000	990.2372216	9.802381409
0.4	995.1391239	4.766248849	0.000000000	990.4697500	9.608672315
0.6	995.2084815	4.653508191	0.000000000	990.6976802	9.418793972
0.8	995.2777056	4.543434297	0.000000000	990.9211064	9.232667638
1.0	995.3467980	4.435959782	0.000000000	991.1401231	9.050214572

Table 2. Numerical approximate solutions for $S_h(t), I_h(t), R_h(t), S_m(t)$ and $I_m(t)$ using the Power series collocation method at $N = 5$.

t	$S_h(t)$	$I_h(t)$	$R_h(t)$	$S_m(t)$	$I_m(t)$
0.2	995.0696306	4.881724503	0.000000000	990.2372205	9.802382231
0.4	995.1391238	4.766251000	0.000000000	990.4697476	9.608674407
0.6	995.2084815	4.653510309	0.000000000	990.6976777	9.418796040
0.8	995.2777056	4.543435041	0.000000000	990.9211055	9.232668378
1.0	995.3467980	4.435959632	0.000000000	991.1401234	9.050214445

Table 3 Numerical approximate solutions for $S_h(t), I_h(t), R_h(t), S_m(t)\ and\ I_m(t)$ using the Hermite collocation method at $N = 3$.

t	$S_h(t)$	$I_h(t)$	$R_h(t)$	$S_m(t)$	$I_m(t)$
0.2	995.0696307	4.881725926	0.000000000	990.2372191	9.802383438
0.4	995.1391259	4.766249493	0.000000000	990.4697501	9.608672264
0.6	995.2084887	4.653500908	0.000000000	990.6976909	9.418784960
0.8	995.2777231	4.543410377	0.000000000	990.9211396	9.232640011
1.0	995.3468324	4.435908103	0.000000000	991.1401936	9.050155898

Table 4. Numerical approximate solutions for $S_h(t), I_h(t), R_h(t), S_m(t)\ and\ I_m(t)$ using the Hermite collocation method at $N = 5$.

t	$S_h(t)$	$I_h(t)$	$R_h(t)$	$S_m(t)$	$I_m(t)$
0.2	995.0697464	4.879324365	0.000000000	990.2405474	9.799610857
0.4	995.1392096	4.764511063	0.000000000	990.4721599	9.606664813
0.6	995.2085096	4.653066439	0.000000000	990.6982939	9.418282793
0.8	995.2776610	4.544698135	0.000000000	990.9193553	9.234126367
1.0	995.3466835	4.438997873	0.000000000	991.1359120	9.053722726

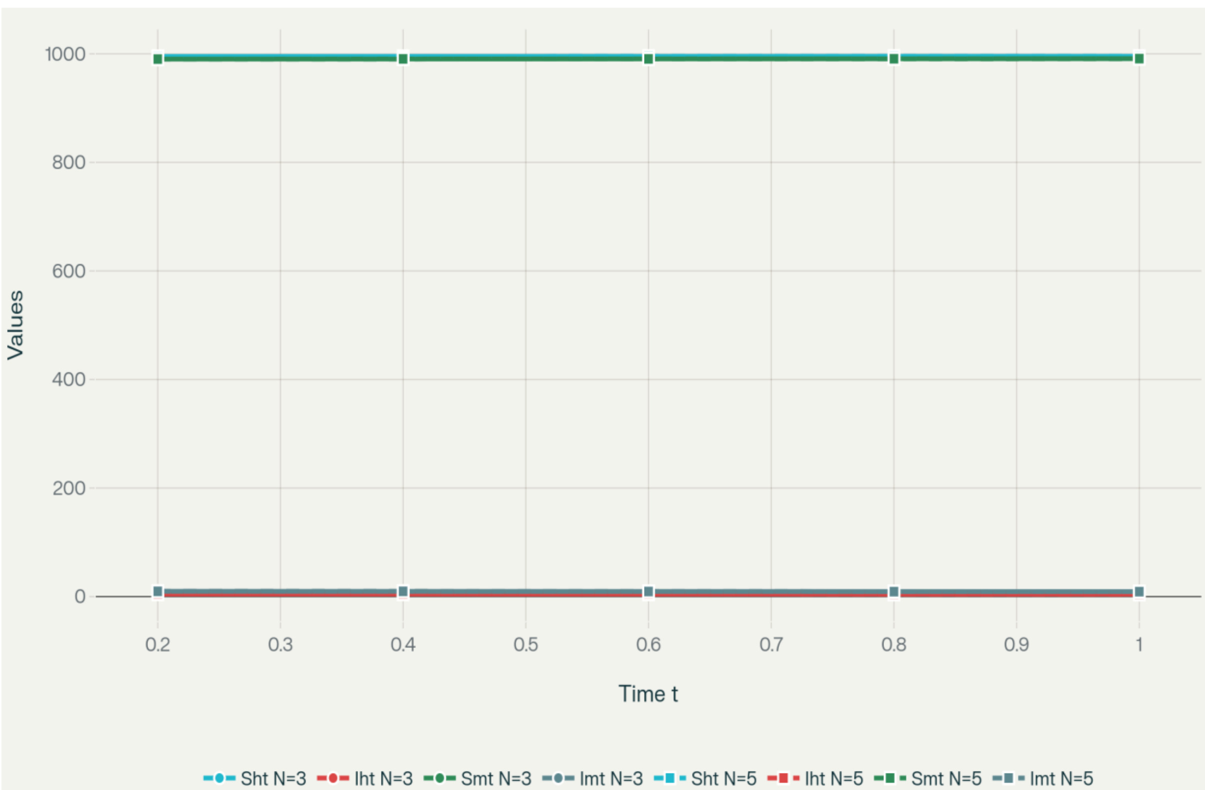

Figure 1. Numerical results of different values of Hermite collocation method.

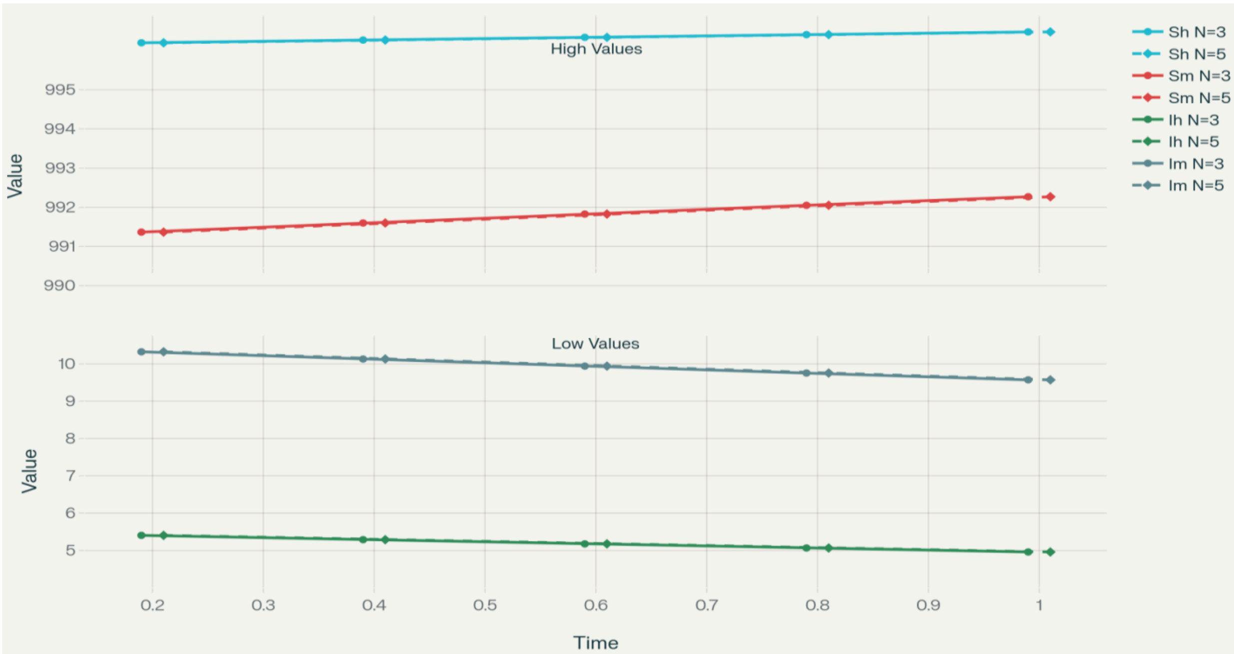

Figure 2. Numerical results of different values of power series collocation method.

Analyzing the disease free equilibrium

The DFE was derived as

$$E_0 = \left(\frac{\Lambda_h}{\mu_h}, 0, 0, \frac{\Lambda_m}{\mu_m}\right)$$

and the basic reproduction number was obtained as

$$R_0 = \sqrt{\frac{\beta_h \, \beta_m \, N_m}{N_h \mu_m (\gamma_h + \mu_h + \delta_h)}}$$

Using the baseline parameters, the computed $R_0 = 2.2361$, indicating that malaria infection can persist in the population in the absence of control measures.

RESULTS AND DISCUSSION

In this study, the malaria model was analyzed numerically, and the following observations were made:

Power series collocation method

The numerical approximate solutions for $S_h(t), I_h(t), R_h(t), S_m(t)$, and $I_m(t)$ using the Power Series collocation method at $N = 3$ and $N = 5$ over the time points $t = 0.2$ to 1.0. We observed that:

i) The solutions for $S_h(t)$, and $S_m(t)$ show a gradual increase over time, with almost no visible difference between $N = 3$ and $N = 5$, indicating numerical stability and convergence of the method.

ii) $I_h(t)$ and $I_m(t)$ both decrease gradually over time, again showing negligible differences between the two values of N. This suggests that the infection variables in the model are well-approximated even at low polynomial degrees.

iii) $R_h(t)$ remains constant at zero throughout, which may correspond to specific problem conditions or assumptions in the model.

iv) The close overlap of the graphs at $N = 3$ and $N = 5$ for all variables indicates that increasing the polynomial degree beyond 3 results in only slight refinements, proving the effectiveness and efficiency of the Power Series collocation method for this problem. The graph demonstrates the stability, accuracy, and convergence of the Power Series collocation method for this numerical problem.

Hermite collocation method

The numerical approximate solutions for $S_h(t), I_h(t), R_h(t), S_m(t)$, and $I_m(t)$ using the Hermite collocation method at $N = 3$ and $N = 5$ over the time points $t = 0.2$ to 1.0.

It was observed that:

i) $S_h(t)$ and $S_m(t)$: Both parameters show a slow, steady increase with very close results for $N = 3$ and $N = 5$, indicating strong consistency and stability in the Hermite collocation method for these variables.
ii) $I_h(t)$: There is a gradual decrease over time, and the values for $N = 3$ and $N = 5$ are nearly identical, reinforcing the precision of the method even with fewer collocation points.
iii) $R_h(t)$: Remains zero throughout the time points for both **N** values, which may represent boundary conditions or a parameter fixed by the problem. Overall, the graph demonstrates the Hermite collocation method's reliable and stable performance in approximating solutions, with little difference between results at $N = 3$ and $N = 5$, implying that even lower-order approximations are effective.

Conclusion

This study presented a numerical approach for approximating the solution of a nonlinear malaria transmission model using the collocation method. Two polynomial bases polynomials were employed to approximate the solutions of the system's differential equations representing human and mosquito populations. The resulting algebraic equations were solved using matrix inversion techniques for each compartment, ensuring numerical stability and computational efficiency.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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